

# Analytical Approach for Lateral Torsional Buckling Strength of Triangular Web Profile Steel Beams

Mohammed Mansour, Sherif M. Ibrahim, Mohamed Korashy

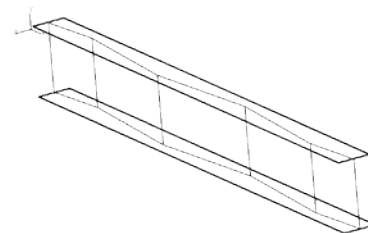
**Abstract-** Triangular web profile (TWP) steel beam is special case of corrugated web profile where web follows seesaw configurations along the beam longitudinal axis. This triangular configuration of web allows minimizing web thickness and eliminate the need for transverse stiffeners. This paper develops a simplified analytical approach to calculate the elastic lateral torsional buckling of TWP steel beam. The elastic lateral torsional buckling is an essential input to estimate the inelastic strength of beam in many international codes such as Euro code. The proposed approach is validated through linear and non-linear three-dimensional finite-element model. The lateral torsional buckling strength of TWP steel beam is calculated and compared to that of flat web beam. Results show that web triangulation increases the lateral torsional strength relative to flat web beam. The paper also investigates the effect of triangulation parameters on the lateral torsional buckling strength in order to maximize it.

**Index Terms-** Triangular web profile, built-up steel I-beam, lateral torsional buckling, Warping constant, Finite Element Analysis.

## 1 INTRODUCTION

Built up I-beams with flat web (FW) profile are widely used as main structural elements in bridges, buildings and many fields because of its favorable properties. In these built-up I-beams, it has been common practice to use more steel material in web although it has little contribution to bending resistance. This results in uneconomical sections as steel material is an expensive material. The use of corrugated web profile in I-girder consistently reduces the out-of-plane web instability and eliminates the use of transverse stiffeners. Using a slender web for deep FW beam is necessary for large spans and large loads, thus, making the web buckling problems more relevant in design. For that reason, the webs in FW beams are usually strengthened with stiffeners. The stiffeners are designed to divide the web into panels supported along the stiffener lines. Stiffeners welding has two disadvantages; the first is reducing fatigue life, and the second is increasing the fabrication cost. Owing to its profiled form, corrugated web (CW) exhibits an enhanced shear stability, bending behavior and hence eliminate the need for transverse stiffeners or thicker web plates. Triangular web profile (TWP) steel beam is modified configuration of CW steel beam. TWP steel beam has a built-up cross section consists of two flanges welded to web plate of triangular corrugation profile as

shown in Fig.1. The main advantage of TWP steel beam is increasing the out-of-plane beam stiffness, therefore, increases the beam lateral torsional buckling strength under in-plane bending, and decreases out-of-plane deformations under any out-of-plane loading. Additional advantages of TWP steel beam include the increase of shear buckling strength without the need of using vertical stiffeners leading to reduction of fabrication cost. This paper develops a simplified approach to calculate the elastic critical lateral torsional buckling strength of TWP steel beams. This approach incorporates the web triangulation along the beam length by deriving a modified equation for the warping constant and utilizing it into the classical equation of elastic lateral torsional buckling (LTB) developed by Timoshenko and Gere [1]. A three-dimensional linear finite element model is used to validate the proposed approach. The elastic critical lateral torsional buckling strength obtained through the proposed approach is used to predict the inelastic strength of TWP steel beams using Euro code equations and the results are compared to those of three dimensional non-linear finite element model. A parametric study to investigate the effect of web triangulation angle ( $\alpha$ ) and triangulation width-to-flange width ratio ( $\gamma$ ) on the lateral torsional buckling strength in comparison with the conventional FW steel beam is also presented.



a- Isometric view of a triangular web profile model

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b- Plan view of a triangular web profile model

Fig. 1: A typical shape of TWP steel beam

## 2 LITERATURE REVIEW

Steel beams with TWP have considerable increase in the out-of-plane stiffness compared to FW steel beams as reported by De’nan, et al. [2] who carried a numerical study using finite element analysis to determine the influence of web triangulation on the elastic behavior of beams and found that the value of major moment of inertia ( $I_x$ ) for the TWP is in a range from 0.754 to 0.818 times that  $I_x$  of FW steel beam while the value of minor moment of inertia ( $I_y$ ) for the TWP is in a range from 1.523 to 1.686 times that  $I_y$  of FW steel beam. In 2012, De’nan and Hashim [3] studied behavior of TWP steel beams and compared it to FW steel beams. They used thin type shell elements to carry out finite element analysis of TWP and FW steel beams. They used two control specimens with two cross sections of dimensions 200×100×6×3 mm and 200×75×5×2 mm. Each cross section was analyzed for different spans of 3000 mm, 4000 mm and 4800 mm long with variable corrugation angles (15°, 30°, 45°, 60° and 75°). It was noted that corrugation angles 45° and 75° have the lowest value of deflection either about minor or major axes of TWP steel beam with other corrugation angles and FW steel beams. This indicates that the TWP steel beam has higher resistance when the web corrugation angle is 45° or 75°. They concluded that TWP steel beam with both corrugation angles 45° or 75° has a higher resistance to bending about minor and major axes. In 2013, De’nan and Hashim [4], conducted an experimental study on the bending behavior about major and minor axes of TWP steel beams FW steel beams. It was concluded that the TWP steel beam has a lower deflection values about minor axis but has higher deflection values about major axis compared to that FW steel beam. In 2015, De’nan and Hazwani [5] performed an experimental study on lateral torsional buckling TWP steel beams. They found that the buckling moment resistance of the TWP steel beam section is greater than that of the FW by 10% to 17%. In 2013, De’nan [6] studied the effect of TWP on the shear behavior of steel I-beam. It was concluded that the TWP section has higher shear capacity compared to the normal FW profile section (mention the enhancement percentage).

TWP steel beam is a special case of CW steel beam. Many researches [7, 8, 9, 10 and 11] developed several equations to calculate the warping constant ( $C_w$ ) of CW steel beams which can be used to determine the elastic critical lateral torsional buckling ( $M_{cr}$ ) using Timoshenko and Gere formula given in Eq. (1).

$$M_{cr} = \frac{\pi}{L_b} \sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w} \quad (1)$$

Where  $L_b$  is the unsupported length of the beam,  $E$  is Young’s modulus,  $G$  is the shear modulus,  $I_y$  is the minor moment of inertia of the beam cross section,  $J$  is the torsional section constant and  $C_w$  is the warping constant.

Lindner [7] investigated some empirical formulas for the warping constant of CW steel beams based on experimental tests results. The study confirmed that the torsional section constant may be calculated using the same method as beam with flat web. Sayed-Ahmed [8] concluded that the resistance of CW steel beams to lateral torsional buckling is higher than those with flat webs by magnitude up to 37%. He proposed an equivalent web thickness ( $t_{weqv}$ ) which is defined in terms of the corrugated web geometrical configurations as given in Eq. (2).

$$t_{weqv} = \frac{a+c}{a+b} t_w \quad (2)$$

where  $a$  and  $b$  are shown as Fig. 2 and  $t_w$  is web thickness of beams.

Sayed-Ahmed [8] used the proposed  $t_{weqv}$  to calculate all the cross section properties ( $I_y$ ,  $J$  and  $C_w$ ) and incorporate them in Eq. (1).

Moon et al. [9] derived an expression, based on the force method, for the warping constant of symmetrical CW steel beam based on ignoring the web contribution. The location of the shear center of cross-section lies at a distance ( $d$ ) outside the corrugated web. They calculated the warping constant based on average corrugation depth ( $d_{av}$ ). They also used a reduced shear modulus proposed by Samanta and Mukhopadhyay [10]. The equation for the modified shear modulus of corrugated web steel beam is given as follows:

$$G_c = \frac{(a+b)}{(a+c)} G \quad (3)$$

Where the term  $(a + b)$  is the projected length of the corrugated plate and the term  $(a + C)$  is the actual length of the corrugated plate as shown in Fig. 2. They suggested an average corrugation depth  $d_{av}$  to take the change in corrugation depth into account as follows

$$d_{av} = \frac{(2a+b)}{2(a+b)} d_{max} \quad (4)$$

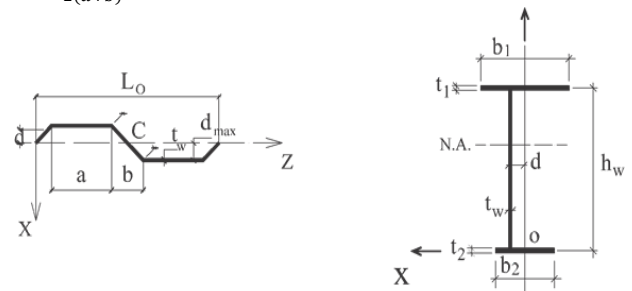


Fig. 2: A Typical shape of CWG web section (Ibrahim [11])

Moon et al [9] suggested to calculate  $C_w$  by integrating the normalized unit warping ( $W_n$ ) curve across the entire cross-section. They ignored web contribution when calculating  $C_w$ . Ibrahim [11] presented an equation for  $C_w$  for symmetrical

CW steel beam that express Moon et al [9] method in final form as follows:

$$C_{w(Moon)} = C_{w(Flat)} + I_w d_{av}^2 \quad (5)$$

Where,  $C_{w(Flat)}$  = warping constant of FW beam =  $\frac{I_y}{4} h_w^2 = \frac{B_f^3 t_f}{24} h_w^2$ ,  $I_w$  = web moment inertia about major axis =  $\frac{t_w h_w^3}{12}$ , and  $h_w$  is the beam web height.

Moon et al. [9] and Ibrahim [11] neglected the web contribution in the calculations of the moment of inertia about y-axis due to the accordion effect.

Nguyen et al. [12] considered the corrugated web to be fully active and took it into consideration in calculating the cross-section static properties and neglected the accordion effect. The shear center is located at a distance  $X_s$  from the center of the two flanges and given as follows:

$$X_s = d + \left( \frac{6B_f t_f}{6B_f t_f + h_w t_w} \right) d = d + k_1 d \quad (6)$$

$$k_1 = \left( \frac{6B_f t_f}{6B_f t_f + h_w t_w} \right) \quad (7)$$

Nguyen et al. [12] calculated an expression for  $C_w$  at a given value of corrugation ( $d$ ) as follows:

$$C_w = \frac{h_w^2}{24} \frac{b t_f}{(6b t_f + h_w t_w)} (6t_f b^3 + b_f^2 h_w t_w + 12h_w t_w d^2) \quad (8)$$

Substituting for  $C_{w(Flat)}$ ,  $I_w$  and  $k_1$ , the warping constant  $C_w$  can be rewritten as

$$C_w = C_{w(Flat)} + K_1 I_w d^2 \quad (9)$$

To account for the cross section change along the beam length, Nguyen et al [12] proposed to calculate the warping constant as the average value at  $d = 0$  and at  $d = d_{max}$ . In this case, the average depth of corrugation would be  $d_{avg} = \frac{d_{max}}{\sqrt{2}}$ . The final form  $C_w(Nguyen)$  can be written as follows:

$$C_w(Nguyen) = C_{w(Flat)} + \frac{k_1}{2} I_w d_{max}^2 \quad (10)$$

The moments of inertia of symmetric CW beam at corrugation at a distance ( $d$ ) from the center of the two flanges were given as follows [12]

$$I_{x,CW} = B_f t_f \frac{h_w^2}{2} + \frac{h_w^3 t_w}{12} \quad (11)$$

$$I_{y,CW} = \frac{B_f t_f (2t_f B_f^3 + B_f^2 h_w t_w + 12h_w t_w d^2)}{6(2B_f t_f + h_w t_w)} = \frac{B_f t_f (2t_f B_f^3 + B_f^2 h_w t_w)}{6(2B_f t_f + h_w t_w)} + \frac{B_f t_f (12h_w t_w d^2)}{6(2B_f t_f + h_w t_w)} \quad (12)$$

Eq. (12) can be simplified and rewritten as follows:

$$I_{y,CW} = \frac{t_f B_f^3}{6} + \frac{2 B_f t_f}{(2B_f t_f + h_w t_w)} (h_w t_w) d^2 = I_{y(Flat)} + A_w k_2 d^2 \quad (13)$$

where  $I_{y(Flat)}$  is the moment of inertia of FW beam about y-axis =  $\frac{t_f B_f^3}{6}$ ,  $A_w$  is the web area =  $h_w t_w$  and

$$k_2 = \frac{2 B_f t_f}{(2B_f t_f + h_w t_w)} \quad (14)$$

Nguyen et al [12] proposed to calculate the moment of inertia about y-axis as the average value at  $d = 0$  and at  $d = d_{max}$  similar to what they proposed for the warping constant.

Ibrahim [11] investigated the  $C_w$  for CW steel beam with unsymmetrical section by integrating warping constant along cross section of corrugation and concluded the following expression

$$C_{w(Ibrahim)} = C_{w(Flat)} + \left( \frac{a + \frac{b}{3}}{a+b} \right) K I_w d_{max}^2 \quad (15)$$

$$\text{where, } K = 4\rho(1 - \rho) + \psi(2\rho - 1)^2 \quad (16)$$

$$\psi = \left( 4 - \frac{3h_w t_w}{A} \right) \quad (17)$$

and  $\rho$  as defined at Fig. 3

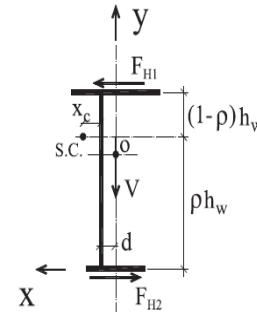


Fig. 3: A typical shape of unsymmetrical section of CWG web (Ibrahim [11])

### 3 ELASTIC LATERAL TORSIONAL BUCKLING OF TWP BEAM

TWP steel beam is a special case from CW steel beam with variable warping constant and variable moment of inertia about the y-axis. Ibrahim [11] proposed that the average value of warping constant for corrugated web girder can be achieved by integrating the variable warping constant along the beam and divide the result by the beam length. The same approach is followed in this research to derive the average values of the warping constant. The integration is performed along the longitudinal axis ( $z$ ) which is shown in Fig. 4. The variable values of the warping constant is given by Eq. (9) in terms of the variable ( $d$ ) which can be rewritten in terms of the triangulation axis variable ( $z$ ). From Fig. 4, the relationship between variable ( $z$ ) and the variable ( $d$ ) can be simply deduced as follows:

$$d = \frac{2 d_{max}}{b} z = \frac{\gamma B_f}{b} z \quad (18)$$

Substituting by the value of  $d$  from Eq. (18) into Eq. (9) and (13)

$$C_{w(TWP)z} = C_{w(Flat)} + I_w k_1 \left( \frac{2 d_{max}}{b} z \right)^2 \quad (19)$$

Integrating the above expressions along half of triangulated part of web equals to  $(0.5 b)$  and divide the result by the same length an average values of the warping constant ( $C_{w(TWP)av}$ ) of TWP beam can be written as follows:

$$C_{w(TWP)av} = \frac{1}{0.5b} \int_0^{0.5b} \left\{ C_{w(Flat)} + I_w k_1 \left( \frac{2 d_{max}}{b} z \right)^2 z^2 \right\} dz \quad (20)$$

$$C_{w(TWP)av} = \left( C_{w(Flat)} + \frac{1}{3} I_w k_1 d_{max}^2 \right) = C_{w(Flat)} + \frac{1}{3} I_w k_1 \left( \frac{\gamma B_f}{2} \right)^2 \quad (21)$$

The reduced shear modulus  $G_{TWP}$  can be calculated from Eq. (3) and considering the flat part term equals to zero, i.e.:  $a = 0$ .

$$G_{TWP} = \frac{b}{c} G = \cos(\alpha) \times G \quad (23)$$

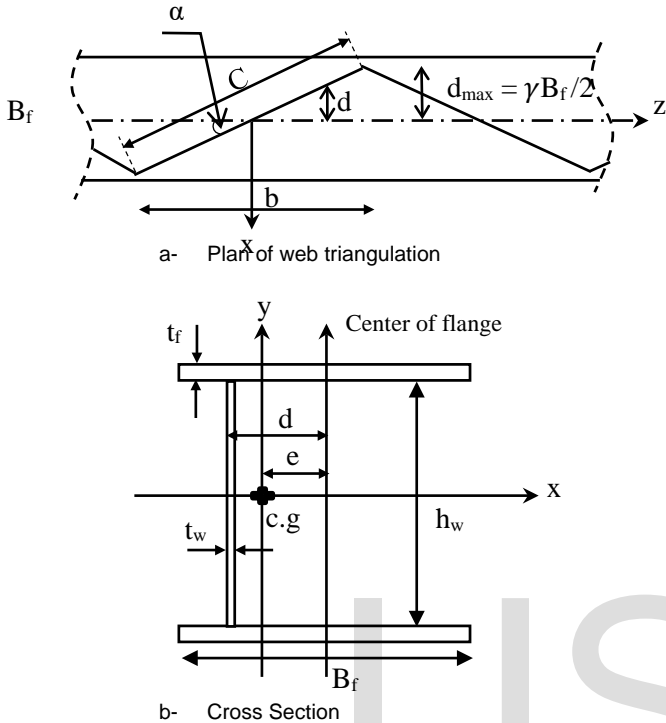


Fig. 4: A typical shape of TWP web section

The average warping constant and reduced shear modulus from Eq. (21 and 23) respectively are utilized to calculate the elastic lateral torsional buckling moment for TWP steel beam subject to uniform bending using Eq. (1) as follows:

$$M_{cr(TWP)} = \frac{\pi}{L_b} \sqrt{EI_{y(Flat)} G_{TWP} J + \left(\frac{\pi E}{L_b}\right)^2 I_{y(Flat)} C_{w(TWP)av}} \quad (24-a)$$

$$M_{cr(TWP)} = \frac{\pi \sqrt{EI_{y(Flat)}}}{L_b} \sqrt{G J \cos \alpha + E \left(\frac{\pi}{L_b}\right)^2 \left(C_{w(Flat)} + \frac{1}{3} I_w k_1 \left(\frac{\gamma B_f}{2}\right)^2\right)} \quad (24-b)$$

It should be noted that the web effect is neglected in moment of inertia about y-axis due to accordion effect as recommended by Moon et al. [9] and Ibrahim [11] and value of  $I_{y(Flat)}$  is utilized in Eq. (24-a and 24-b). The deduced expression for the critical elastic moment includes the effect of triangulation angle ( $\alpha$ ) and the triangulation width-to-flange width ratio ( $\gamma$ ).

#### 4 FINITE ELEMENT MODELING OF TWP STEEL BEAMS

In order to verify the accuracy of using the average warping constant and the reduced shear modulus of TWP steel beam

to estimate the elastic critical lateral torsional buckling moment, a comparison with finite element (F.E.) analysis is performed. Finite element software package ABAQUS V16.4. [13] is utilized to construct numerical model of TWP steel beams. Thin shell element was chosen to represent the flanges and the web of the beam model. In this research, two types of thin shell elements which are quadrilateral thin shell element (QSL8) and triangular thin shell element (TSL6) are used; as shown in Fig. 5. The material properties for all modelled specimens are selected with the following values: Young's modulus,  $E = 2.09 \times 10^5$  N/mm<sup>2</sup>, shear modulus,  $G = 79 \times 10^3$  N/mm<sup>2</sup>, Poisson's ratio = 0.3 and yield strength = 355 N/mm<sup>2</sup>. Boundary conditions of the beam in this study is hinged at one side and roller for the other side to simulate a simply supported beam. Two equal end moments are applied at the ends as a loading condition by applying two eccentric loads at each end as shown in Fig 6. The finite element analysis of each beam is conducted through two phases. The first phase is a linear buckling analysis, which is performed on a beam having perfect geometry in order to obtain the probable elastic lateral torsional buckling mode. From this linear buckling analysis, the elastic critical lateral torsional buckling moment is obtained by solving the associated Eigenvalue problem. Typical buckling mode of TWP steel beam is shown in Fig. 7. The second phase of the finite element analysis is the non-linear analysis of the based on buckled beam mode shape obtained from the first step. This phase is performed to obtain the ultimate capacity of the initially imperfect beam as well as predicting the lateral deformations and failure mode of the examined beams. Both material and geometrical non-linearities are incorporated in the non-linear static analysis which is performed by using the Modified Newton-Raphson (MNR) technique and employing the Arc-length control throughout the solution routine of the parametric study. Initial imperfection of a value equals to beam span /1000 is considered in the non-linear analysis while material non-linearities are taken into consideration by introducing a bilinear stress-strain curve for steel.

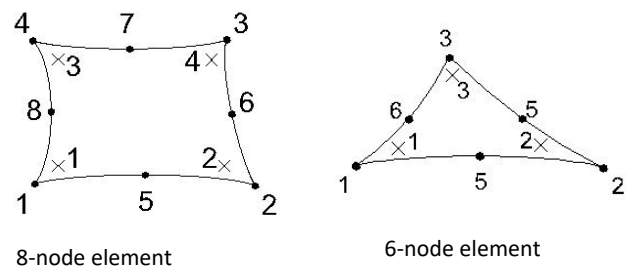


Fig. 5: A typical shape of shell element



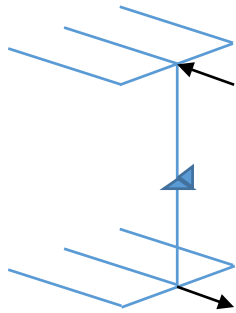


Fig. 6: A typical shape of boundary conditions

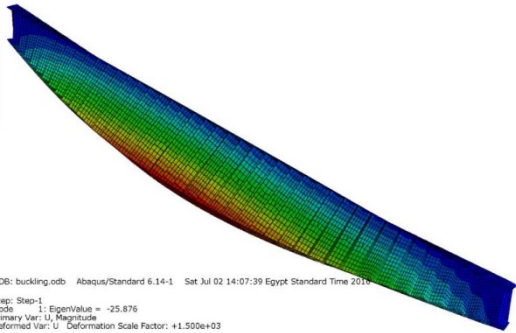


Fig. 7: A typical shape of buckling mode

#### 4.1 Verification of Elastic Lateral Torsional Buckling Moment of TWP Steel Beam

Sixteen TWP steel beams specimens are analyzed using the linear buckling finite element analysis. The cross section and beam span are presented in Table (1). The notations ( $h_w$ ,  $B_f$ ,  $t_f$  and  $t_w$ ) indicated in Table (1) are similar to those shown in Fig. 4. The values of elastic lateral torsional moment ( $M_{cr}$ ) from theoretical Eq. (24) using the proposed average warping constant in Eq. (21) are compared to those obtained from finite element model. Moreover, the elastic critical lateral torsional moment is also compared to those obtained from Moon et al [9], Ibrahim [11] and Nguyen et al [12]. The comparison between results is shown in Table (2). It is evident from Table (2) that there is excellent agreement between F.E results and the current study with the deduced ( $C_{w(TWP)}$ ) and the reduced shear modulus for TWP with maximum difference between them of merely 5.3% and most of current study values are on the conservative side with respect to the finite element analysis. On the other hand, there is a significant difference between F.E results and results of Nguyen et al. [12] as they do not consider the accordion effect in web which would lead to an overestimate of the moment of inertia about Y-axis specially for cross sections with large  $\gamma$  values. The current study provide more accurate results with the finite element analysis compared to the results of Moon et al. [9] and Ibrahim [11].

TABLE 1 DIMENSIONS AND PARAMETERS STUDIED OF TWP BEAMS

Cross section $h_w$ - $B_f$ - $t_w$ - $t_f$	Span L (m)	$\alpha$	$\gamma$
900-300-8-10	12	45°	0.2-0.4-0.6-0.8
	15	45°	0.2-0.4-0.6-0.8
1200-300-8-10	12	45°	0.2-0.4-0.6-0.8
	15	45°	0.2-0.4-0.6-0.8

TABLE 2 COMPARISON BETWEEN DIFFERENT EQUATIONS AND FINITE ELEMENT ANALYSIS ( $M_{CR(TWP)} / M_{CR(FINITE ELEMENT)}$ )

Cross section $h_w$ - $B_f$ - $t_w$ - $t_f$	$\gamma$	Moon et al. [9]	Ibrahim [11]	Nguyen et al [12]	Current Study
900-300-8-10 Span = 12 m	0.2	0.942	0.944	1.041	0.992
	0.4	0.922	0.928	1.075	0.972
	0.6	0.924	0.937	1.168	0.977
	0.8	0.943	0.965	1.311	0.999
900-300-8-10 Span = 15 m	0.2	0.916	0.917	1.017	0.985
	0.4	0.885	0.891	1.037	0.952
	0.6	0.879	0.891	1.114	0.947
	0.8	0.892	0.911	1.243	0.962
1200-300-8-10 Span = 12 m	0.2	0.964	0.966	1.050	1.005
	0.4	0.962	0.970	1.112	1.002
	0.6	0.980	0.999	1.236	1.021
	0.8	1.014	1.045	1.417	1.056
1200-300-8-10 Span = 15 m	0.2	0.944	0.946	1.032	1.002
	0.4	0.933	0.941	1.082	0.989
	0.6	0.943	0.960	1.192	0.999
	0.8	0.972	1.001	1.360	1.028

#### 4.2 Verification of the Inelastic Lateral Torsional Buckling Moment of TWP Steel Beam

The inelastic lateral torsional buckling moment for TWP steel beams can be estimated using the rules of Eurocode EC-3, Part 1-3 [14] and utilizing the elastic critical lateral torsional buckling moment obtained based on the proposed  $C_{w(TWP)}$ . The moment resistance  $M_{Rd}$  for fully effective flanges is given according to the EC-3 as follows:

$$M_{Rd} = \frac{W_n \chi_{LT} F_y}{\gamma_M} \tag{25}$$

where  $W_n = W_p =$  plastic section modulus for class 2 cross sections.

$$W_p = B_f t_f (h_w + t_f) + t_w \frac{h_w^2}{4} \quad (26)$$

$$\chi_{LT} = \frac{1}{\varphi_{LT} + \sqrt{\varphi_{LT}^2 - \beta \lambda_{LT}^2}} \leq \begin{cases} 1 \\ \frac{1}{\lambda_{LT}^2} \end{cases} \quad (27)$$

$\gamma_M$  is partial factor representing resistance of member and is set to unity, and

$F_y$  is the yield strength of steel

$$\varphi_{LT} = 0.5 [ 1 + \alpha_{LT} (\lambda_{LT} - \lambda_{LT,0}) + \beta \lambda_{LT}^2 ] \quad (28)$$

$$\lambda_{LT} = \sqrt{\frac{M_p}{M_{cr(TWP)}}} \quad (29)$$

where  $\alpha_{LT}$  is an imperfection factor which is considered in this study as 0.76 for buckling curve (d). For all models considered in this research, the value of  $h_w/B_f > 2.0$ ; hence buckling curve (d) is utilized.  $\beta$  and  $\lambda_{LT,0}$  are parameters depending on the type of beam section, and EC 3 [14] recommends values of  $\beta = 0.75$  and  $\lambda_{LT,0} = 0.4$ . The inelastic lateral torsional buckling moment obtained from Eq. (25) which is based on  $M_{cr(TWP)}$  obtained from Eq. (24-b) for the two cross sections shown in Table 1 for the 12 m span is compared to non-linear F.E. analysis results as shown in Fig. 8 and 9. It is evident from these figures that there is excellent agreement between the nonlinear F.E. analysis results and theoretical ones obtained based on the modified  $C_w(TWP)$  with maximum difference between them of merely 6%.

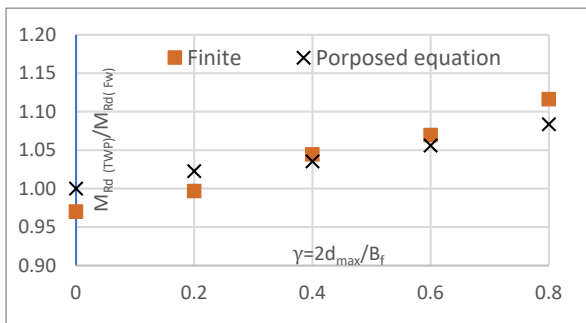


Fig. 8: Comparison between proposed equation and finite element for section 900-300-8-10 mm with span 12m

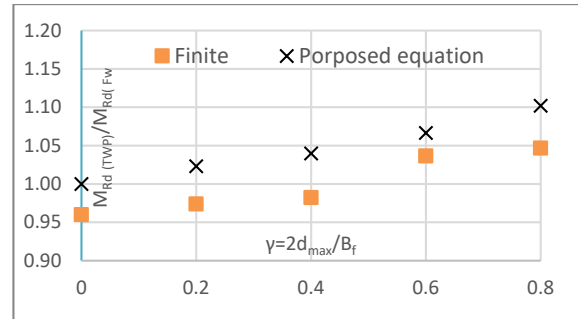


Fig. 9: Comparison between proposed equation and finite element for section 1200-300-8-10 mm with span 12m

## 5 RESULTS AND DISCUSSION

A parametric study on TWP steel beams is conducted in order to determine effect of geometric parameters on the critical lateral torsional buckling moment  $M_{cr(TWP)}$ . The main parameters that affect the  $M_{cr(TWP)}$  are: i) web corrugation angle ( $\alpha$ ) and ii) width of web corrugation-to-flange width ( $\gamma = \frac{2d_{max}}{B_f}$ ). The values of all these parameters are indicated on Table (3). A Total of 48 TWP steel beams are included in this parametric study. The cross-section dimensions ( $h_w, B_f, t_f$  and  $t_w$ ) are similar to those used for the verification of the simplified analytical approach. The value of  $M_{cr(TWP)}$  for each beam is calculated and normalized by dividing it by the critical lateral torsional moment of the corresponding FW beam ( $M_{cr(FW)}$ ) which is given by Eq. (1).

TABLE 3: DIMENSIONS AND PARAMETERS STUDIED OF TWP BEAMS

Cross section hw-Bf-tw-tf	Span L (m)	$\alpha$	$\gamma$
900-300-8-10	12	15°, 30°, 45°	0.2-0.4-0.6-0.8
	15	15°, 30°, 45°	0.2-0.4-0.6-0.8
1200-300-8-10	12	15°, 30°, 45°	0.2-0.4-0.6-0.8
	15	15°, 30°, 45°	0.2-0.4-0.6-0.8

### 5.1 Effect of Web Corrugation Angle ( $\alpha$ ) on Lateral Torsional Buckling Moment

To investigate the effect of change the web corrugation angle on bending behavior about major axis, TWP steel beams with three corrugation angles ( $\alpha = 15^\circ, 30^\circ$  and  $45^\circ$ ) with two cross sections 900-300-8-10 and 1200-300-8-10 and span  $L=12m$  are analyzed. The results are shown in Fig. 10 to 13.

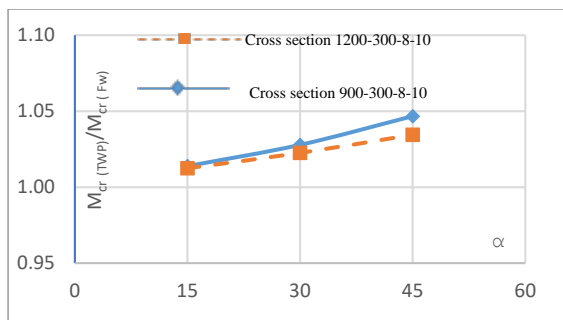


Fig. 10: ( $M_{cr(TWP)} / M_{cr(FW)}$ ) with different  $\alpha$  for  $\gamma=0.2$

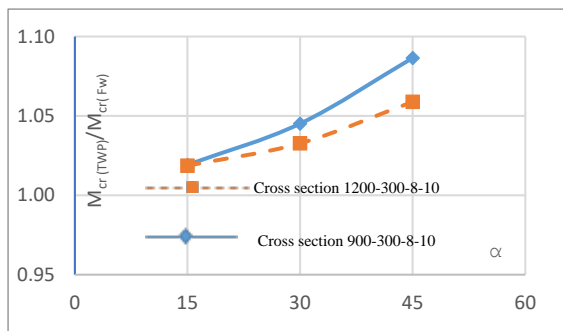


Fig. 11: ( $M_{cr(TWP)} / M_{cr(FW)}$ ) with different  $\alpha$  for  $\gamma=0.4$

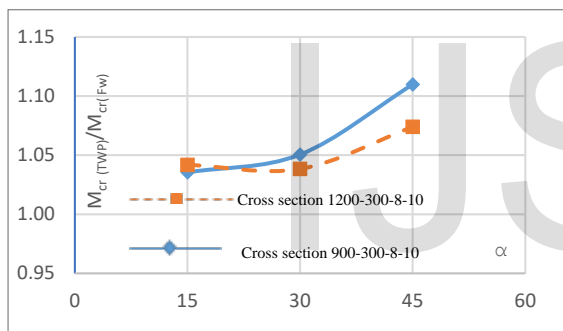


Fig. 12: ( $M_{cr(TWP)} / M_{cr(FW)}$ ) with different  $\alpha$  for  $\gamma=0.6$

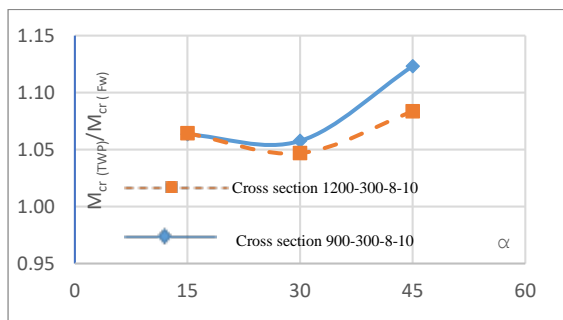


Fig. 13: ( $M_{cr(TWP)} / M_{cr(FW)}$ ) with different  $\alpha$  for  $\gamma=0.8$

Results show that generally the LTB resistance of the TWP steel beam is stronger than that of the FW beam. In this numerical study, it was noted that LTB resistance of 45° web corrugations angle is the highest LTB resistance value of TWP steel beam with maximum increase of 13.5% compared to FW steel beam. This is because, as the corrugation angle increases, the number of slanting webs increases throughout the length, leading to increase LTB resistance. While for the

corrugation angle 15°, TWP steel beam has only a maximum increase of 6% compared to FW steel beam.

### Effect of Web Corrugation Width-to-Flange Width ratio (g) on Lateral Torsional Buckling Moment

To investigate the effect of the web corrugation width-to-flange width ratio on the bending behavior, TWP steel beam models with g varies from 0.2 to 0.8 are investigated. Results for the two investigated cross sections and with span L=12 m and 15m are shown in Fig. 14 to 17.

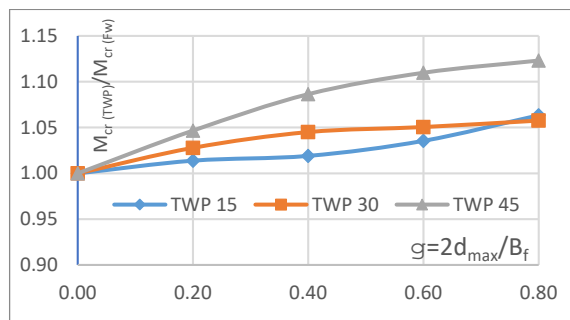


Fig. 14: ( $M_{cr(TWP)} / M_{cr(FW)}$ ) with different g for section 900-8-300-10 with span 12m.

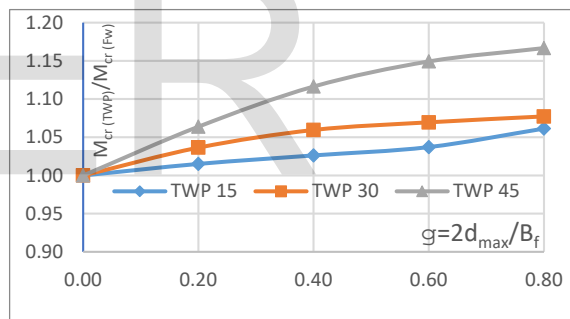


Fig. 15: ( $M_{cr(TWP)} / M_{cr(FW)}$ ) with different g for section 900-8-300-10 with span 15m.

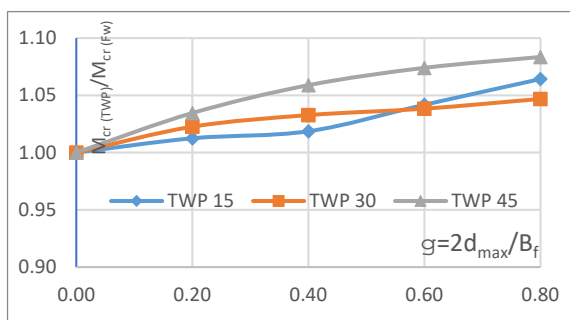


Fig. 16: ( $M_{cr(TWP)} / M_{cr(FW)}$ ) with different g for section 1200-8-300-10 with span 12m.

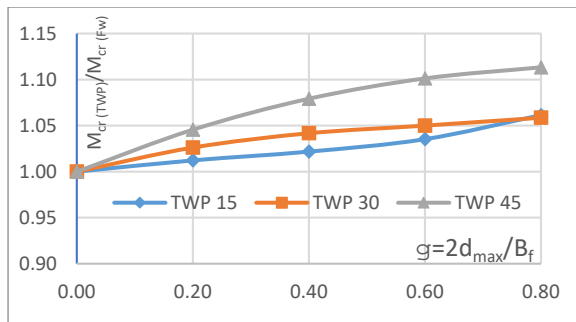


Fig. 17: ( $M_{cr(TWP)} / M_{cr(FW)}$ ) with different  $g$  for section 1200-8-300-10 with span 15 m.

It can be noted that maximum rate of increase of  $M_{cr(TWP)}$  occurs at value of  $g = 0.2$ , while the rate of increase of  $M_{cr(TWP)}$  drops significantly for  $g$  between 0.6 and 0.8. Thus, for a practical configuration of TWP steel beams the values with  $g$  larger 0.6 would not result in significant increase the critical moment except for triangulation angle of 15°.

## 6 SUMMARY AND CONCLUSIONS

In this paper, a simplified analytical approach is proposed to calculate the critical lateral torsional buckling moment TWP steel web beams based on modified values of the warping constant. The proposed approach is verified through linear buckling analysis conducted by general-purpose F.E. program (ABAQUS) and the comparison proved excellent agreement. The proposed approach can also be extended to estimate the inelastic ultimate moment of TWP steel beams by applying the rule of Eurocode 3 which also proved to be in excellent agreement with non-linear F.F. results. A numerical parametric study was carried out for simply supported TWP steel beam to investigate the influence of the web triangulation angle and the triangulation width-to-flange width ratio on the critical moment. The study showed that TWP steel beams have significant increase in the lateral torsional buckling failure mode compared to FW steel beams. A maximum increase in the critical moment is observed when the corrugation angle is 45°. The maximum rate of increase in the critical moment is observed at corrugation width-to-flange width ratio ( $g$ ) of 0.2 and no significant increase in the critical is observed by for values of  $g$  larger than 0.6.

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